

Static and Dynamic Behavior of Unsaturated Soils Using the Theory of Mixtures with Interfaces

Kanthasamy K. Muraleetharan and Changfu Wei

School of Civil Engineering and Environmental Science, University of Oklahoma,
Norman, OK73019, USA
muralee@ou.edu, wei-1@ou.edu

ABSTRACT

The theory of mixtures with interfaces (TMI) is used to develop field equations governing the behavior of unsaturated porous media subjected to static and dynamic loading. The interfaces existing between the bulk phases in unsaturated porous media are considered as independent phases in the TMI. The derived governing equations and an elastoplastic constitutive model are implemented into a finite element computer code, U_DYSAC2. Static and dynamic behavior of an unsaturated soil embankment is simulated using U_DYSAC2 and is compared to the saturated soil behavior. Influence of soil skeleton volume changes occurring during loading on soil suction is highlighted and the unsaturated soil embankment is shown to behave more rigidly.

INTRODUCTION

The increasing use of compacted soils in earthquake prone areas, such as in landfill clay liners, has created a necessity for better understanding of the dynamic behavior of these unsaturated soils. Unsaturated soils are three-phase porous media consisting of a solid skeleton, a pore liquid, and a pore gas. A fully coupled analysis of an unsaturated soil requires a procedure that can rigorously incorporate the interactions among the three bulk phases and the interfaces existing between them.

The analyses of the static and dynamic behavior of unsaturated soils are generally performed based on various extensions of Biot's saturated porous media theories (Biot 1962), for example, Li and Zienkiewicz (1992) and Schrefler and Zhan (1993). In these procedures, the momentum transfers between the bulk phases are approximated by Darcy's type expression, and a generalized effective stress formulation is used to describe the stress-strain behavior of the soils. However, theoretical and experimental research show that the application of the effective stress concept in describing the behavior of unsaturated soils is limited, see, for example, Fredlund and Rahardjo (1993).

Based on the theory of mixture with interfaces (TMI), Muraleetharan and Wei (1999a) presented a theoretical model for unsaturated porous media. In TMI the interfaces between the bulk phases are considered as independent phases. Muraleetharan and Wei (1999a) also showed that obtaining an explicit expression for the effective stress in unsaturated soils is impossible and that the pressure differences between various bulk phases are thermodynamically constrained. These constrained relationships in fact represent the so-called closure equations in unsaturated soil mechanics. The derived

governing equations have been implemented into a finite element computer code U_DYSAC2 (Muraleetharan and Wei 1999b). The governing equations based on TMI and spatially discretized equations are first summarized and then an example analysis of an unsaturated soil embankment using U_DYSAC2 is presented and compared with saturated soil behavior.

GOVERNING EQUATIONS

The mass balance equations are given by

$$\frac{D^\alpha}{Dt}(n^\alpha \rho^\alpha) + n^\alpha \rho^\alpha (\nabla \cdot \mathbf{v}^\alpha) = \sum_{\beta \neq \alpha} \hat{e}_{\alpha\beta}^\alpha \quad (1)$$

where ρ^α denotes the real density of phase α , and $\hat{e}_{\alpha\beta}^\alpha$ the exchange rate of mass through interface $\alpha\beta$ into α -phase. Neglecting the mass exchange between the solid and the fluids, (1) is subjected to $-\hat{e}_{\ell g}^\ell - \hat{e}_{\ell g}^g = 0$ and $\hat{e}_{s\alpha}^s = \hat{e}_{s\alpha}^\alpha = 0$ ($\alpha = \ell, g$). Linear momentum balance equation of the mixture is given by

$$\rho \frac{D^s \mathbf{v}^s}{Dt} + n^\ell \rho^\ell \left(\frac{D^s \mathbf{v}^{\ell,s}}{Dt} + \mathbf{v}^{\ell,s} \nabla \cdot \mathbf{v}^\ell \right) + n^g \rho^g \left(\frac{D^s \mathbf{v}^{g,s}}{Dt} + \mathbf{v}^{g,s} \nabla \cdot \mathbf{v}^g \right) - \nabla \cdot \boldsymbol{\sigma} - \rho \mathbf{g} = 0 \quad (2)$$

where $\rho = \sum n^\alpha \rho^\alpha$ is the total density of the mixture, $\mathbf{v}^{\alpha,s}$ ($\alpha = \ell, g$) is the relative velocity of α -phase with respect to the solid skeleton, $\boldsymbol{\sigma}$ is the total stress tensor, and \mathbf{g} is the gravitational acceleration tensor. Finally, the general flow equations are given by

$$\rho^\alpha \nabla G^\alpha - \rho^\alpha \mathbf{g} + \rho^\alpha \frac{D^\alpha \mathbf{v}^\alpha}{Dt} = -\frac{1}{n^\alpha} \boldsymbol{\mu}^\alpha \cdot \mathbf{v}^{\alpha,s}, \quad \alpha = \ell, g \quad (3)$$

where G^α is the Gibbs' thermodynamic potential and is defined as $G^\alpha = A^\alpha + p^\alpha / \rho^\alpha$; A^α is the Helmholtz free energy and p^α is the thermodynamic pressure of α -phase. If it is assumed that the dependence of A^α on the interfacial area densities and volume fractions can be neglected, Eq. (3) reduces to the classical form (see, for example, Li and Zienkiewicz 1992, Schrefler and Zhan, 1993). The solid phase can be assumed to be incompressible for practical purposes ($\rho^s = \text{constant}$) and the only required closure equation is given by

$$n^\ell = f(\varepsilon_v, p_c) \quad (4)$$

where ε_v is the volumetric strain of the solid skeleton and $p^c (= p^g - p^\ell)$ is the soil suction.

Muraleetharan and Nedunuri (1998) recently developed an elastoplastic bounding surface constitutive model for unsaturated soils. This model developed in terms of two stress state variables net stress (total stress minus pore gas pressure) and suction can be described as follows

$$\boldsymbol{\sigma}^N = \boldsymbol{\sigma} - p^g \mathbf{I} = \mathbf{D} : \boldsymbol{\varepsilon} \quad (5)$$

where $\boldsymbol{\varepsilon}$ is the Lagrangian strain tensor of the soil skeleton and $\boldsymbol{\sigma}^N$ is the net stress tensor.

In the finite element solution procedure, the displacement of the soil skeleton, the pore water pressure, and the pore air pressure are used as primary unknowns. These unknowns can be approximated as $u_i = N_A^s \bar{u}_{Ai}$, $p^\ell = N_B^\ell \bar{p}_B^\ell$, and $p^g = N_C^g \bar{p}_C^g$. Here, upper case character A, B, C denote the node number; i denotes the spatial direction; N_A^α ($\alpha = s, \ell, g$) represents the shape function at node point A associated with the spatial discretization of the corresponding primary unknown. Utilizing the Galerkin's method the matrix form of governing equations can be derived as follows

$$\mathbf{M}_{uu} \ddot{\mathbf{u}} + \mathbf{K}_{ug} \bar{\mathbf{p}}^g + \int_{\Omega} N_{A,j}^s \boldsymbol{\sigma}_{ij}^N d\Omega = \mathbf{F}_u \quad (6a)$$

$$\mathbf{M}_{\ell u} \ddot{\mathbf{u}} + \mathbf{C}_{\ell u} \dot{\mathbf{u}} + \mathbf{C}_{\ell\ell} \dot{\bar{\mathbf{p}}}^\ell + \mathbf{C}_{\ell g} \dot{\bar{\mathbf{p}}}^g + \mathbf{K}_{\ell\ell} \bar{\mathbf{p}}^\ell = \mathbf{F}_\ell \quad (6b)$$

$$\mathbf{M}_{gu} \ddot{\mathbf{u}} + \mathbf{C}_{gu} \dot{\mathbf{u}} + \mathbf{C}_{g\ell} \dot{\bar{\mathbf{p}}}^\ell + \mathbf{C}_{gg} \dot{\bar{\mathbf{p}}}^g + \mathbf{K}_{gg} \bar{\mathbf{p}}^g = \mathbf{F}_g \quad (6c)$$

The spatially discretized form (6) and the elastoplastic constitutive relationship (5) have been incorporated into a computer code U_DYSAC2 (Muraleetharan and Wei 1999b). It is worthy to note that when the soil becomes saturated (6) collapses into its saturated counterpart written in the u-p format and the transition between saturated and unsaturated states is automatic and smooth. This feature is very desirable for the finite element solution of a problem involving both saturated and unsaturated soils.

EXAMPLE ANALYSIS

In order to illustrate the capabilities of U_DYSAC2 an example analysis is presented here. In this example a centrifuge model constructed of compacted speswhite kaolin is first spun up from 1 g to 80 g in the centrifuge and then a base shaking is applied to the model. The prototype dimensions of the model and the finite element mesh used are shown in Fig. 1. Initial degree of saturation of the soil is 88% and the initial soil suction is 102 kPa. The initial pore water and pore air pressures were set to -102 kPa and 0 kPa (atmospheric), respectively, to reflect the initial suction of 102 kPa. Both the spin up and the shaking were simulated using U_DYSAC2 by appropriately choosing the time integration parameters. Although all the analyses were carried out in the model scale, results are presented here in the prototype scale. Vertical displacement of the selected nodes and pore fluid pressure variations within two elements during spin up are presented in Fig. 2. As expected elements those experienced larger increase in total stress (near the centerline and at the bottom of the embankment) underwent larger volumetric strain and

consequently larger change in degree of saturation and suction. For example, Elements 14 and 69 experienced a volumetric strain of -4.313×10^{-3} and -3.959×10^{-3} , respectively, and the degree of saturation became 89.1% and 88.6%, respectively, at 320 seconds.

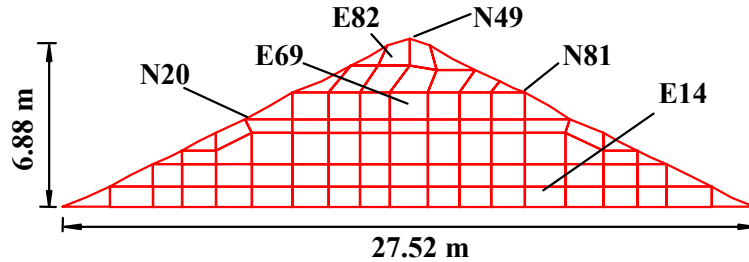


Figure 1. Finite Element Mesh of the Unsaturated Kaolin Embankment

Using the initial state obtained at 320 seconds a dynamic analysis was carried out by applying the base motion shown in Fig. 3. Vertical displacement of the crest, pore fluid pressure changes within Elements 14 and 69 and horizontal acceleration near the crest are also shown in Fig. 3. Also shown in Fig. 3 is the vertical crest settlement and horizontal acceleration near the crest obtained for a corresponding saturated soil embankment (Muraleetharan et al. 1994). Although the horizontal acceleration values near the crest are essentially the same in saturated and unsaturated embankments, crest settlement in the unsaturated embankment is half as much as that of the saturated case. Stiffening of the soil caused by soil suction is the reason for the reduced settlement of the unsaturated soil embankment. Substantially larger increase in degree of saturation was observed during shaking than during spin up. These increases and the corresponding reduction in suction seen in Fig. 3 are consistent with the cyclic loading induced volumetric strains during shaking. For example, Elements 14 and 69 experienced a volumetric strain of -4.520×10^{-2} and -1.935×10^{-2} , respectively, and the degree of saturation values became 93.6% and 91.0%, respectively, around 13 seconds. More cyclic pore fluid pressures can be seen at Element 14 compared to Element 69. A higher initial static shear stress in Element 14 is the reason for subsequent higher cyclic pore fluid pressures during shaking. These results indicate the importance of taking into account the change in volumetric strain and its influence on soil suction through Eq. (4) during static and dynamic loading events. The results presented in Figs. 2 and 3 also show the capabilities of U_DYSAC2 in simulating the complex behavior of unsaturated soils during static and dynamic events.

ACKNOWLEDGEMENTS

This research was supported by a grant (Grant No. CMS 9501718) from the National Science Foundation (NSF). The cognizant NSF program director for the grant is Dr. Clifford J. Astill.

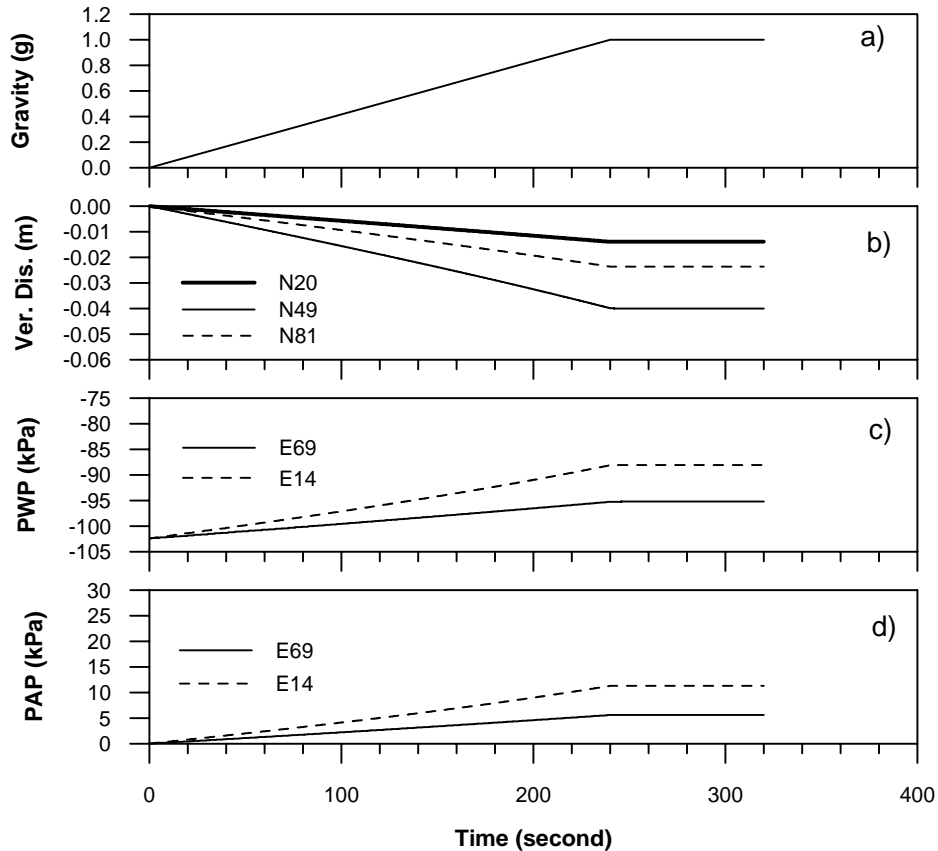


Figure 2. Vertical Displacements, Pore Water Pressures (PWP), and Pore Air Pressures (PAP) During the Static Analysis Using U_DYSAC2

REFERENCES

- Biot, M. A. (1962). "Mechanics of deformation and acoustic propagation in porous media", *J. Appl. Phys.*, 33(1), 1482-1498.
- Fredlund, D. G. and Rahardjo, H. (1993). *Soil Mechanics for Unsaturated Soils*, John Wiley & Sons, New York.
- Li, X., and Zienkiewicz, O.C. (1992). "Multiphase flow in deforming porous media and finite element solutions." *Computers and Structures*, 45(2), 211-227.
- Muraleetharan, K. K., Mish, K.D., and Arulanandan, K. (1994). "A fully coupled non-linear dynamic analysis procedure and its verification using centrifuge test results." *Int. J. Numer. Anal. Mech. Geomech.*, 18, 305-325.
- Muraleetharan, K.K., and Nedunuri, P.R. (1998). "A bounding surface elastoplastic constitutive model for monotonic and cyclic behavior of unsaturated soils." *Proceedings, 12th Engineering Mechanics Conference*, ASCE, La Jolla, CA, 1331-1334.

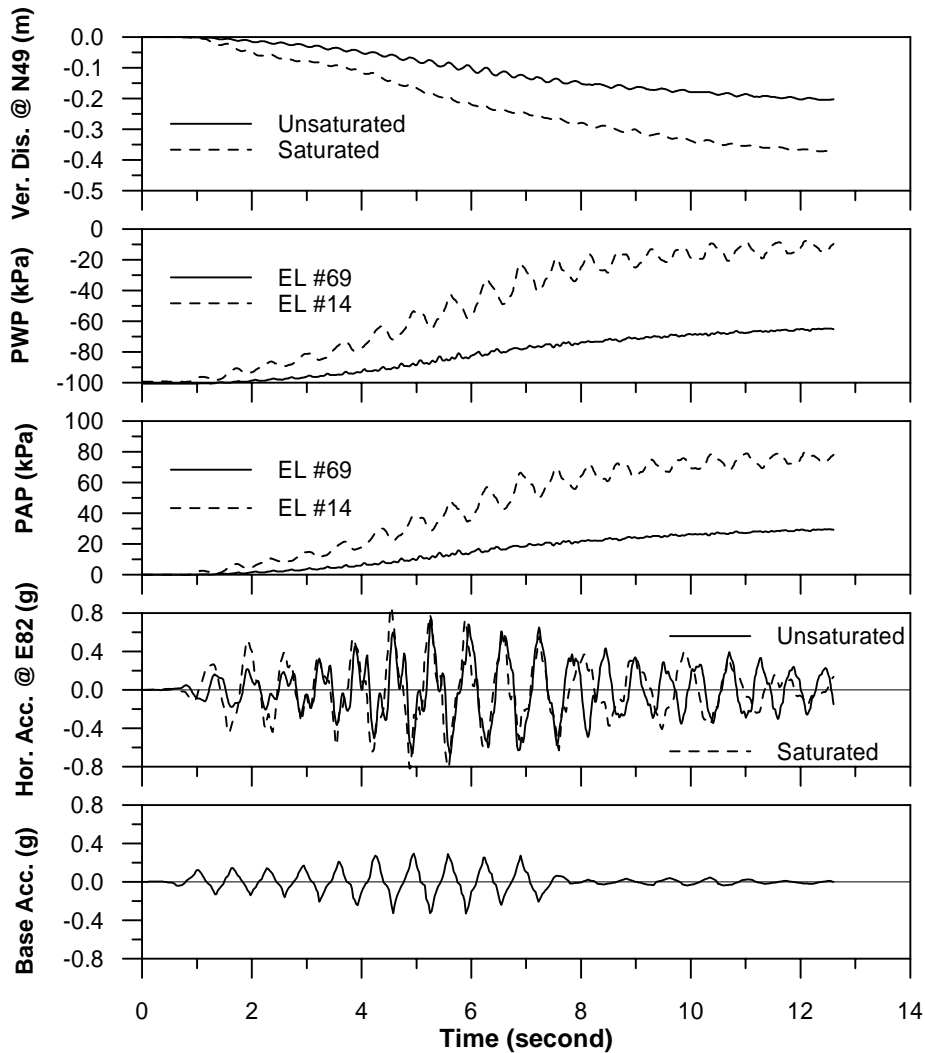


Figure 3. Vertical Crest Displacement, Pore Water Pressures (PWP), Pore Air Pressures (PAP), and Horizontal Crest Acceleration-Time Histories During the Dynamic Analysis.

Muraleetharan, K.K., and Wei, C.-F. (1999a). "Dynamic behavior of unsaturated porous media : Governing equations using the theory of mixtures with interfaces (TMI)." *Int. J. Numer. Anal. Meth. Geomech.* 23, 1579-1608.

Muraleetharan, K.K., and Wei, C.-F. (1999b). "U_DYSAC2: Unsaturated Dynamic Soil Analysis Code for 2-dimensional problems." *Computer Code*, School of Civil Engineering and Environmental Science, University of Oklahoma, Norman, Oklahoma, October.

Schrefler, B. A., and Zhan, X. (1993). "A fully coupled model for water flow and airflow in deformable porous media." *Water Resources Research*, 29, 155-167.